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Robust Controller Design for Uncertain Systems

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Abstract— ***This paper proposes a simple approach to design a PID controller for an uncertain SISO system that will guarantee closed loop stability when the input is varied between 0 and 60 % of its working range. The Robust Controller design method used here is based on Small Gain theorem. A family of transfer functions corresponding to different working ranges is used to describe the uncertain plant for which the controller is to be designed. In this paper, the controller is designed for a warm air-drying chamber which is a good example for an uncertain system. The design is carried out considering the plant behaviour at 3 working points. The closed loop step responses obtained, prove the robust operation of the designed controller for the plant under the considered working range.***

Keywords— **Robust controller, Uncertain model, Small gain theorem, Closed loop stability.**

I. Introduction

Any physical system can never be described by an exact mathematical model. The mathematical model is always only an approximation of the true system dynamics since we ignore certain factors such as non-linearities, time delay, effects of reduced order model, variations in system parameters due to environmental changes etc. which may affect the plant behaviour. This results in modelling errors and hence improper design of a controller.

The deviation of the actual plant model from the nominal one is called model uncertainty. Uncertainty can be classified as disturbance signals and dynamic perturbations. The former includes input and output disturbances and the latter represents discrepancy between the mathematical model and the actual dynamics of the system.

Unstructured uncertainty:

Dynamic perturbations such as due to unmodelled high frequency dynamics can be lumped together into a single perturbation block Δ. This is called unstructured uncertainty. The unstructured dynamics uncertainty can be represented in different ways as shown in figures 1 and 2. $G_p(s)$ denotes the actual perturbed system dynamics and $G_o(s)$ denotes the nominal model of the system. The block Δ is uncertain but is normally bounded. The additive uncertainty representations give an account of the absolute error between the actual dynamics and the nominal model. The multiplicative uncertainty representations indicate the relative errors.

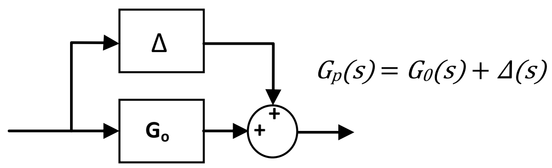

Fig.1 Additive Perturbation Model

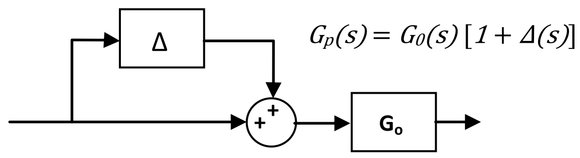

Fig.2 Multiplicative Perturbation Model

Structured uncertainty:

An uncertain model can be described mathematically by a transfer function having uncertain parameters as its coefficients. These coefficients vary about their nominal values within the uncertainty bounds under different working conditions.

Parametric uncertainty:

In many systems dynamic perturbations may also be caused by inaccurate description of component characteristics. Such perturbations may be represented by variations of certain system parameters over some ranges. These affect the low frequency range performance and are called parametric uncertainties [i].

PID controllers are widely popular in industries because they are easy to implement. However they lack generality since they use heuristic tuning methods like Ziegler-Nichols [ii]. Dubravska M. and Harsanyi L. [iii, iv] have proposed a novel and easy to use method of designing a PI/PID controller for uncertain systems using Small gain theorem. Ying J. Huang and Yuan-Jay Wang [v] have demonstrated the design of a robust PID controller for an uncertain plant with bounded parameters using Kharitonov theorem. Clarke et al. [vi] introduced the Generalized predictive control to overcome the limitations of Model predictive control like open loop system being unstable, unknown plant orders etc. The computational burden has been significantly reduced by the GPC technique for FOPDT model proposed by Bordons and Camacho [vii, viii]. A control problem is formulated as a mathematical optimization problem in H_∞ technique [ix, x, xi]. This paper

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aims to use a simple, frequency domain based approach to design a PID controller for a third order uncertain system that is described by a family of Nyquist plots.

II. Methodology

Consider a SISO uncertain system $G(s)$ described by a family of transfer functions and being controlled by a controller $C(s)$. Then the loop transfer function is given by $L(s) = C(s)\,G(s)$.

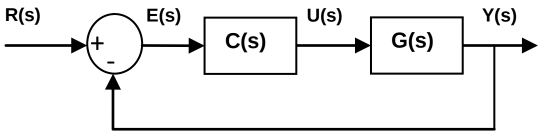

Fig.3 Closed Loop Control System

According to Small Gain theorem, if the magnitude of the loop gain is less than one then, the closed loop system is robustly stable. Hence for closed loop robust stability, $|L(s)| < 1$ *for all* $\omega \in [0, \infty]$

Suppose that the plant to be controlled is uncertain and described by a number of stable transfer functions. Let its nominal transfer function be $G_o(s)$. Then the perturbed system can be represented in the form of an unstructured additive uncertainty as shown in fig. 4. $W_a(s)$ represents the weighting transfer function and $\Delta(s)$ represents a set of transfer functions with peak magnitudes less than or equal to one for all frequencies.

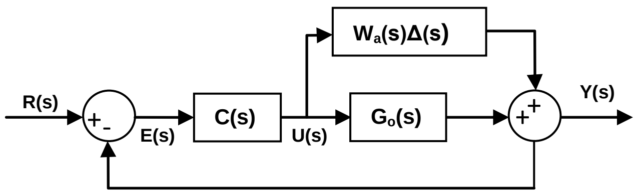

Fig.4 Closed Loop Uncertain System

$$G(s) = G_0(s) + W_a(s)\,.\,\Delta(s) \qquad (1)$$

if $|\Delta(s)| \leq 1$ *then,* $|W_a(s)\,.\,\Delta(s)| \leq |W_a(s)|$

The weighting transfer function is chosen such that the following condition is satisfied.

$|W_a(s)| \geq Max\,|G_k(s)\text{-}G_0(s)|$ *for all* $\omega \in [0, \infty]$

where k = 1, 2, 3, ... n (2)

where $G_k(s)$ represents the set of n stable transfer functions describing the uncertain system.

The characteristic equation of the closed loop uncertain system is given by, $1 + C(s)\,G(s) = 0$

Using equation (1) in the above equation we have,

$$1 + C(s)\,[G_0(s) + W_a(s)\,.\,\Delta(s)] = 0$$

$$[1 + C(s)G_o(s)]\left[1 + \frac{C(s)G_o(s)}{1 + C(s)G_o(s)}.\frac{W_a(s).\Delta(s)}{G_o(s)}\right] = 0$$

The Closed Loop Nominal transfer function is given by,

$$G_{NCL}(s) = C(s)\,G_0(s) \,/\, [1 + C(s)\,G_0(s)] \qquad (3)$$

$$[1 + C(s)\,G_0(s)]\,[1 + G_{NCL}(s).W_a(s)\,\Delta(s)\,/\,G_0(s)] = 0 \qquad (4)$$

Let us assume that the nominal closed loop system is stable. This implies that the nominal characteristic equation, $1 + C(s)\,G_0(s) = 0$ is stable.

The Necessary and Sufficient condition for the closed loop system to be stable is that equation (4) should be stable which implies that $1 + C(s)\,G_0(s) = 0$ should be stable and $[1 + G_{NCL}(s).W_a(s)\,\Delta(s)/G_0(s)] = 0$ must satisfy Small Gain theorem. i.e., the magnitude of the loop gain must be less than one.

$|G_{NCL}(s).W_a(s)\,\Delta(s)\,/\,G_0(s)| < 1$ *for all* $\omega \in [0, \infty]$

Consider the worst case value of $\Delta(s)$ i.e., $|\Delta(s)| = 1$ *for all* $\omega \in [0, \infty]$ in the above equation.

$$|G_{NCL}(s).W_a(s)\,/\,G_0(s)| < 1$$

$$|G_{NCL}(s)| < |G_0(s)\,/\,W_a(s)| \qquad (5)$$

Hence we need to make a suitable choice of $G_{NCL}(s)$ so that the above condition is satisfied.

Consider (3).

$$C(s)\,G_0(s)\,/\,[1 + C(s)\,G_0(s)] = G_{NCL}(s)$$

$$C(s) = G_{NCL}(s)\,/\,\{\,G_0(s)\,[1 - G_{NCL}(s)]\,\}$$

Each of the transfer functions in the above equation are expressed as ratios of pertinent polynomials.

$$\frac{C_c(s)}{C_m(s)} = \frac{\dfrac{G_{NCLc}(s)}{G_{NCLm}(s)}}{\dfrac{G_{oc}(s)}{G_{om}(s)}\left[1 - \dfrac{G_{NCLc}(s)}{G_{NCLm}(s)}\right]}$$

The Controller transfer function is given by,

$$C_c(s)/C_m(s) = G_{om}(s)\,G_{NCLc}(s)/\{G_{0c}(s)[G_{NCLm}(s) - G_{NCLc}(s)]\} \qquad (6)$$

Expressing the transfer functions in equation (3) as ratios of pertinent polynomials,

$$G_{NCLc}/G_{NCLm} = [C_c\,G_{oc}/C_m G_{om}]/[1 + C_c\,G_{oc}/C_m G_{om}]$$

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$G_{NCLc}(s)/G_{NCLm}(s) = G_{0c}(s)/[C_m(s)\,G_{0m}(s)/C_c(s) + G_{0c}(s)]$

$G_{NCLc}(s)/G_{NCLm}(s) = G_{0c}(s)/[C_m(s)\,P_{0m}(s) + G_{0c}(s)]$

Equating the numerators and denominators on both sides,

$$G_{NCLc}(s) = G_{0c}(s) \quad (7)$$

$$G_{NCLm}(s) = C_m(s)\,P_{0m}(s) + G_{0c}(s) \quad (8)$$

where $P_{0m}(s) = G_{0m}(s)\,/\,C_c(s)$ (9)

Using (7) in (6) the controller transfer function is given by,

$C_c(s)/C_m(s) = G_{om}(s)\,/\,[G_{NCLm}(s) - G_{0c}(s)]$

The general transfer function for a PID controller is given by,

$C_c(s)/C_m(s) = N(s)/K\,s = (n_d s^2 + n_p s + n_i)/K\,s$

We choose the numerator and the denominator of the controller transfer function as follows.

From (9), $G_{om}(s) = P_{0m}(s)\,C_c(s)$

From (8), $G_{NCLm}(s) - G_{0c}(s) = P_{0m}(s)C_m(s)$

Hence, the controller transfer function becomes,

$G_{om}(s)/[G_{NCLm}(s) - G_{0c}(s)] = P_{0m}(s)C_c(s)/P_{0m}(s)C_m(s)$

Equating this to the general transfer function for a PID controller,

$G_{om}(s)/[G_{NCLm}(s) - G_{0c}(s)] = P_{0m}(s)N(s)/P_{0m}(s).K.s$

Therefore, we have,

$$G_{om}(s) = P_{0m}(s)N(s) = P_{0m}(s)(n_d s^2 + n_p s + n_i) \quad (10)$$

$$G_{NCLm}(s) - G_{0c}(s) = P_{0m}(s)\,.\,K\,.\,s \quad (11)$$

In (10), we choose the degree of $P_{0m}(s)$ such that it is equal to the degree of $G_{om}(s)$ minus the degree of $N(s)$. Hence $P_{om}(s)$ and $N(s)$ can be determined.

From (7) and (11),

$$G_{NCLc}(s)\,/\,G_{NCLm}(s) = G_{0c}(s)/[P_{0m}(s).K.s + G_{0c}(s)] \quad (12)$$

Now we have expressed G_{NCL} as a function of K. That value of K is chosen so as to satisfy the condition given by equation (5).

Once K is known, the controller parameters can be determined as below.

$$K_d = n_d/K, \quad K_p = n_p/K \quad and \quad K_i = n_i/K \quad (13)$$

III. Design of Robust Controller

The transfer functions for the Warm air-drying chamber [xii] for step change in power from 0-20%, 20-40% and 40-60% of the range are given by,

$G(s) = 10.6/(12893\,s^3 + 8058\,s^2 + 226.7s + 1)$ for 0-20%

$G(s) = 10.5/(2078\,s^3 + 10346\,s^2 + 230.9s + 1)$ for 20-40%

$G(s) = 11.4/(40184\,s^3 + 10506\,s^2 + 255.6s + 1)$ for 40-60%

The Nominal model is given by,

$G_0(s) = 10.833/(18385\,s^3 + 9636.67\,s^2 + 237.73s + 1)$

$= G_{0c}(s)/\,G_{0m}(s)$

Let the PID controller to be designed be given by,

$C(s) = C_c(s)/C_m(s) = (n_d s^2 + n_p s + n_i)/K\,s$

Using equation (10), the PID controller parameters are determined as given below.

$$G_{om}(s) = P_{0m}(s)\,(n_d s^2 + n_p s + n_i) \quad (14)$$

The degree of $P_{om}(s)$ has to be equal to one. Let $P_{om}(s) = p_1 s + 1$

Equating the coefficients of respective powers of s on both sides of equation (14),

$$n_d = 9163.743; \quad n_p = 235.724; \quad n_i = 1; \quad p_1 = 2.0063 \quad (15)$$

Using equation (2), we choose a suitable W_a.

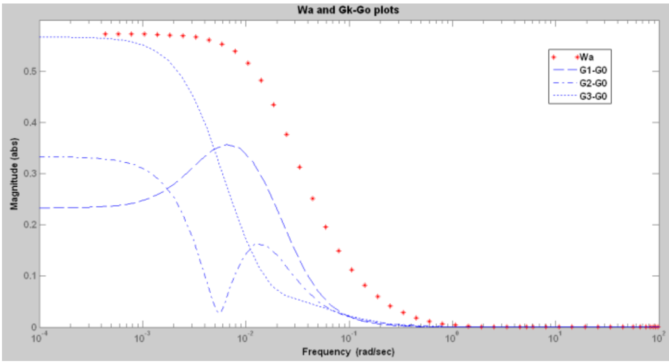

Fig. 5 Frequency Response plots of $|W_a|$, $|G_1 - G_0|$, $|G_2 - G_0|$ and $|G_3 - G_0|$

$|G_0/W_a|$ and G_{NCL} for different K values are plotted. That K value is chosen such that the robust stability condition of equation (5) is satisfied. We have K=120.

Using this K value and equation (15) in equation (13), the controller parameters are determined.

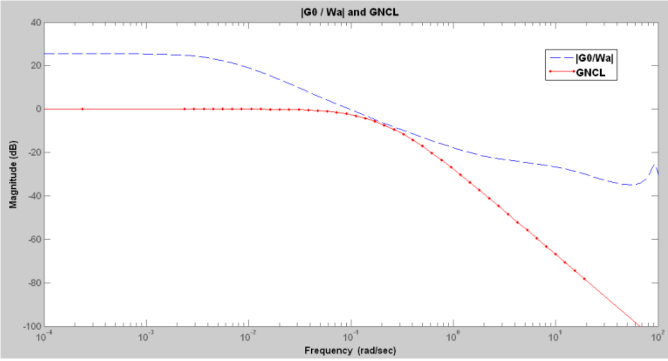

Fig. 6 Frequency Response plot of |G0 / Wa| and G_{NCL}

IV. Results

Fig.8 shows the closed loop step response plot with the introduction of the controller. It can be seen that the designed PID controller results in the Robust operation of the plant under 0-60% of the working range.

Working range	Peak value		Overshoot (%)		Rise time (seconds)		Settling time (seconds)	
	Before	After	Before	After	Before	After	Before	After
0-20%	1.199	1	30.473	0	36.601	18.829	276.473	48.017
20-40%	1.213	1.012	32.000	1.155	41.752	25.380	323.358	39.993
40-60%	1.264	1.010	37.535	0.994	37.489	16.940	373.362	26.068

Table 1. Time response specifications before & after the introduction of the controller

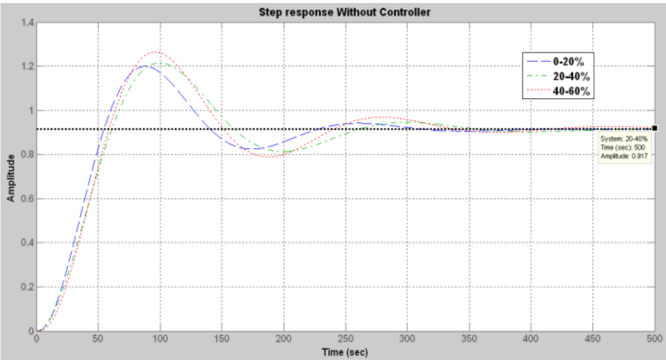

Fig.7 Closed Loop Step response without Controller

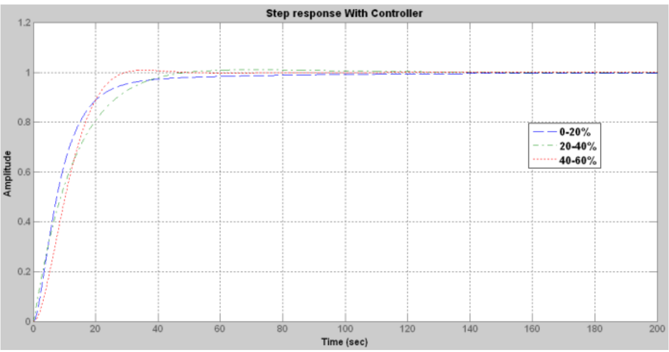

Fig.8 Closed Loop Step response with Controller

With the introduction of the PID Controller, the maximum peak overshoot, maximum rise time and maximum settling time are reduced to 1.155%, 25.38 seconds and 48.017 seconds respectively. Also the steady state error is reduced from 0.917 to 0 which indicates an improved performance.

V. Conclusion

The PID controller design methodology which is simple and easy to use can be successfully applied to a warm-air drying chamber which is an example for an uncertain system. The results show that the designed PID controller results in Robust operation when the temperature is changed between 0 and 60 % of the maximum working range for the three operating points.

References

i. *D.W. Gu, P. Hr. Petkov and M. M. Konstantinov, Robust Control Design with Matlab, Springer –Verlag Inc., London, England, 2005.*

ii. *J. G. Zeigler and N. B Nichols, "Optimum settings for automatic controllers", Transactions ASME, Vol. 64, pp. 759-768, 1942.*

iii. *Harsanyi, L. Dubravska M.: Robust Controller Design for Linear Systems with Parametric and Dynamic Uncertainties, J. Electrical Engineering,52 No. 9-10, pp. 307-310, 2001.*

iv. *Maria Dubravska and Ladislav Harsanyi: Control of Uncertain systems, Journal of Electrical Engineering, Vol. 58, No. 4, pp. 228-231, 2007.*

v. *Ying J. Huang and Yuan-Jay Wang, "Robust PID tuning strategy for uncertain plants based on the Kharitonov theorem", ISA Transactions 39, pp. 419-431, 2000.*

vi. *D. W. Clarke, C. Mohtadi and P.S Tuffs, "Generalized Predictive Control-Part I. The Basic Algorithm", Automatica, Vol. 23, No. 2, pp. 137-148, 1987.*

vii. *E. F. Camacho and C. Bordons, "Implementation of self-tuning generalized predictive controllers for the process industry", International Journal of Adaptive control and signal processing, Vol. 7, pp. 63-73, 1993.*

viii. *Carlos Bordons and Eduardo F. Camacho, "A generalized predictive controller for a wide class of industrial processes", IEEE Transactions on Control Systems Technology, Vol. 6, No. 3, 1998.*

ix. *Bhattacharyya, S. P. et al.: Robust control. The parametric approach, Prentice Hall, 1995.*

x. *John C. Doyle et al., "State space solutions to standard H_2 and H_∞ control problems", IEEE Transactions on Automatic Control, Vol. 34, No. 8, 1989.*

xi. *Kanti B. Datta and Vijay V. Patel, "H_∞ based synthesis for a Robust controller of Interval plants", Automatica, Vol. 32, No. 11, pp. 1575-1579, 1996.*

xii. *Jan Danko, Magdalena Ondrovicova and Vojtech Vesely, "Robust Controller Design to control a Warm air-drying chamber", Journal of Electrical Engineing, Vol. 55, No. 7-8, pp. 207-211, 2004.*